

SPATIAL-TEMPORAL MODEL OF INSECT GROWTH, DIFFUSION AND DERIVATIVE PRICING

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Insect derivatives represent an important innovation in specialty crop risk management. An active over-the-counter market in insect derivatives will require a transparent pricing method. This paper develops an econometric model of the spatio-temporal process underlying a particular insect population and develops a pricing model based on this process. We show that insect derivatives can play an important risk management role in mitigating *B. tabaci* (whitefly) damage in cotton. Beyond developing a new risk management instrument, the key methodological contribution of this paper lies in pricing derivatives with stochastic properties in both space and time dimensions.

Key words: invasive species, derivatives, spatial econometrics, risk management.

Derivative securities are used for risk management purposes in a number of industries. Derivatives, as they are generically called, are financial instruments that assume a value based on an underlying price or index. Stock options are one type of derivative that allows the holder to purchase the underlying asset at a fixed (strike) price. Therefore, a stock option will have a positive intrinsic value if the realized price is higher (lower) than the agreed strike level for a call (put) option. In nonfinancial markets, firms that may be harmed by temperatures that are either too hot or too cold over a particular season are beginning to use weather derivatives in order to mitigate the financial risk that results. Derivatives can also be written on other random processes occurring in nature such as hurricanes or insect populations. This study concerns the latter case—derivatives written on invasive species of all types, and insects in particular.

While there have been no recorded trades to the best of our knowledge, these “insect derivatives” promise to be of particular in-

terest to a diverse set of agricultural firms—farmers, processors, brokers or input suppliers. Insect derivatives are instruments that assume a value if a reference insect population rises above an agreed upon level at a particular place and time. As such, growers who practice integrated pest management (IPM) methods in addition to purchasing a derivative contract can use the resulting increase in value to offset potential losses to their crop. On the other hand, growers who use conventional suppression technologies can offset the rise in cost required to control greater insect concentrations. In either case, chemical companies or other agribusinesses who stand to benefit from rising insect numbers serve as natural counterparties, or agents who will willingly take the other side of the derivative “bet” in order to offset their potential loss in revenue should an infestation not materialize during a particular crop year.

Both the federal government and growers themselves recognize the need for a more effective means of managing the financial risk from invasive species. In a survey of New York State specialty crop growers, White, Uva, and Chenge (2003) report that pests in general represent the third most important source of risk to growers’ net income, ranking only behind adverse temperature and drought. Emergency expenditures by the Animal Plant Health Inspection Service (APHIS) of USDA on eradication programs attests both to the economic significance and immediacy of this problem. While annual spending on emergency eradication was \$10.4 million per year from 1991 to 1995, the annual average between 1999 and

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2001 had risen to over \$230.0 million (USDA 2005a). Indeed, some estimates of the total annual economic loss due to invasive species range as high as \$137.0 billion, including all direct and indirect economic costs (Pimentel et al. 2000).

Increasing levels of government-funded research presumes a failure in the market for invasive species risk. However, the absence of a market for insect risk need not call for a government solution.¹ Rather, insect derivatives represent a market-based solution that will allow growers to optimally choose the number and type of securities to buy in order to transfer the financial risk due to invasive species onto a counterparty. Despite their appeal, there are many institutional barriers that must be overcome before an efficient market for insect derivatives can arise, not the least of which is an agreed pricing mechanism. Another is the basis risk—both spatial and technological—that is inherent in any derivative based on a “real” index value. Third, contract terms would have to be developed to overcome the moral hazard issues that would arise if a grower were left to count his own insects. Therefore, the objective of this study is to show how equilibrium prices for insect derivatives can be derived from well-established risk-neutral pricing methods. We also use this pricing model to achieve a second objective, namely to demonstrate the value of insect derivatives as risk management tools to a wide variety of agricultural producers.

Implementing a risk-neutral pricing model first requires specification and estimation of the stochastic process governing insect diffusion. Unlike prices of financial assets, the process followed by an insect population evolves over both space and time. Consequently, this study represents a significant contribution in the econometrics of financial assets in that we consider both dimensions in estimating the underlying diffusion process. Further, developing a pricing model for insect derivatives also represents a significant advance in option valuation. Pricing models for other financial derivatives are unidimensional, meaning that prices for financial assets flow from benefits that vary only with time. Our concept of insect derivatives, on the other hand, are priced in two-dimensions, as the underlying “asset” (the in-

sect population) confers economic damages on growers that vary both through space and time. In this paper, we use recent developments in option valuation theory to price contingent claims on insect populations that evolve geographically and temporally using field-trial data of *Bemisia tabaci* (whitefly) populations in California cotton (Naranjo, Chu, and Henneberry 1996).

The first section presents a spatial-temporal bioeconometric model of insect population growth and diffusion. Synthesizing current research in spatial econometric and bioeconometrics, we develop this model in sufficiently general terms that its applicability beyond the particular case at hand should be clear. The next section describes a risk-neutral valuation model that is used to price insect derivatives in both space and time. In the third section, we present a simple representative farm value maximization model that incorporates econometric estimates of the relationship between cotton yield, temperature, and insect densities. We use this model to demonstrate the potential effectiveness of insect derivatives in managing insect-infestation risk. A brief description of the *B. tabaci* data used in our empirical application follows. A fifth section presents the econometric estimates and simulation results from both the pricing and representative farm models. The final section offers some general conclusions for how an insect-derivative risk management program may be implemented and draws several implications for how spatial-temporal derivatives may be used more generally in other risk management contexts.

The Model

Bioeconomic models of insect population growth typically consider temporal variation in organism density, or changes in the number of insects per unit of area over time. However, insect populations tend to vary both over time and by geographic location. Entomologists recognize that invasions of new pest populations tend to follow a three-phase process: (1) arrival, (2) establishment, and (3) spread (Hof 1998). The statistical model of population growth and dispersion used here accounts for both temporal and spatial variation by synthesizing empirical research in entomology and spatial econometrics (Anselin and Florax 1995; Gelfand 1998; Anselin 2002). In this section, we describe a bioeconometric model that takes each of these factors into account and allows

¹ Multiple-peril crop insurance (MPCI) can cover insect damage that results in a measurable loss of yield. However, insect-damage consists of reductions in quality, cosmetic appearance, plant-growth or the financial costs of spraying. Many specialty crop growers either do not have access to MPCI or find that it does not suit their particular purposes.

for stochastic variation about the expected path of insect dispersion over space and time.

Bioeconometric Model of Insect Population Growth

In the spatial dimension, insects in one place can influence those at another in one of two ways, depending on whether the effect is direct or indirect: (1) a *spatial-lag* process wherein the population at one point has a direct, causal relationship with the population at another point, or (2) by a *spatial-error* process in which only the random, or stochastic part of population growth at two points is correlated. Areas that are densely populated tend to promote more active reproduction than would otherwise be the case, so the size of the population at point A is likely to be directly related to the population at point B. Therefore, a spatial-lag model is appropriate, but the unobservable factors that promote or hinder growth at A are also likely to be the same as those at B, so a spatial-error model also has some merit. Because the growth process also implies that populations at each location in space are related over time, we include temporal components as well. Incorporating both dimensions into a statistical model for the population of a given pest at a point in space s at time t , the number of insects is written as: $B(s, t)$, $s = 1, 2, \dots, K$ and $t = 1, 2, \dots, N$ with an error process $v(s, t)$ where both B and v follow nonlinear spatial-temporal autoregressive processes.

The data generating processes for both insects and errors are written as general functions of distance in space and time and a set of explanatory variables: $B(s, t) = f(\theta | \mathbf{S}, \mathbf{T}, \mathbf{X})$ where θ is a set of parameters and \mathbf{S} and \mathbf{T} are spatial and temporal weight matrices, respectively. These matrices are specified exogenously and defined in greater detail below. In its most basic form, the insect population-growth-and-diffusion model is given by:

$$(1) \quad B(s, t) = f(\theta | \mathbf{S}, \mathbf{T}, \mathbf{X}) + v(s, t) \\ v(s, t) = h(\Omega | \mathbf{S}, \mathbf{T}) + \varepsilon$$

where \mathbf{X} consists of other exogenous factors (control activities, cooling-degree days (CDD), rainfall, and a binary indicator for the sample year), $v(s, t)$ is assumed to be additive, Ω is a set of parameters for the error process and ε is an iid error term. Equation (1) implicitly assumes nonseparable temporal and spatial effects in order to capture the fact that the spatial interaction among insects changes over

time. This assumption is tested in the empirical application that follows, but is well grounded in theoretical models of insect diffusion.

Consider the evolution of a given insect population over time, conditional on a fixed location, s_0 . Insect numbers are constrained by several biological factors: (1) growth rates depend on the number of adults available to reproduce, (2) reproduction takes time, (3) the environment has a finite capacity to support insect populations, and (4) control activities, typically through insecticides, tend to be quite effective in reducing population counts. Given these facts, entomologists typically model population growth in terms of the logistic function that is common to many other bioeconomic growth models (Clark 1990):

$$(2) \quad \frac{dB(s_0, t)}{dt} = \alpha B(s_0, t) \left(1 - \frac{B(s_0, t)}{K} \right)$$

for the insect population (B) growing at a rate α in an environment with carrying capacity K . The differential equation (2) can be solved for the expected population level at any time, t , which provides a convenient expression for the mean insect population:

$$(3) \quad B^t(s_0, t) = \left(\frac{K}{1 + de^{\mathbf{T}k}} \right) + \mathbf{X}\beta$$

where k is a constant vector, and d represents the starting population value relative to carrying capacity: $(K - B_0)/B_0$. With this specification, $\mathbf{T}k$ represents an absolute measure of the temporal distance of each observation from all others. Depending on data availability, the population may also be a function of temperature, rainfall, annual fixed-effects, host plant abundance, other nonchemical abatement efforts, or predator population. In the *B. tabaci* example, we have data on the first three of these factors so include the vector \mathbf{X} in (3) in a linear form with parameter vector β .

It is well understood, however, that the number of insects measured at a particular point may consist not only of natives, but from others that have migrated to that spot, or have at least bred with others at adjacent locations. For this reason, models of insect population dynamics must take into account spatial diffusion and spillover effects as well. Entomologists have found that the spatial dispersion of insect populations from an initial point of arrival tends to follow a process akin to Fick's Law from particle physics (Liebhold, Halverson, and Elmes 1992; Hof 1998). Applied to

insect diffusion, Fick's Law implies that the number of insects starting at point $s_0 = 0$ will be normally distributed according to the distribution function:

$$(4) \quad B^s(s, t) = \left(\frac{B(s_0, t_0)e^{-(S_k)^2/4\gamma T_k}}{2\sqrt{\pi\gamma T_k}} \right)$$

for any spatial distance, S_k , from the starting point where γ is the diffusion coefficient, which is assumed to be constant for a particular species of insect and geographic location. According to Fick's Law, the variance of any insect population at a point in time, therefore, is γT_k . Notice that spatial diffusion is also a function of time, so clearly the spatial and temporal dimensions of the problem are nonseparable in a structural sense. Therefore, equations (3) and (4) provide a basis for the nonlinear components of the insect growth model. Let the combined effect of space and time on the mean insect level be defined as $B^m(s, t) = B^s(s, t) + B^t(s_0, t)$ so that the full nonlinear model is written as:

$$(5) \quad B(s, t) = \mathbf{X}\beta + \left(\frac{K}{1 + de^{T_k}} \right) + \left(\frac{B(s_0, t_0)e^{-(S_k)^2/4\gamma T_k}}{2\sqrt{\pi\gamma T_k}} \right) + u(s, t)$$

where $u(s, t)$ is a random error term for any point in space and time (s, t) . Anselin (2002), however, suggests that spatial data may exhibit spatial-lag patterns even after controlling for theoretical relationships such as that hypothesized here. Pace et al. (2000) extend this logic to spatial-temporal data. In the empirical application, we also estimate a composite version of (5) that allows us to test whether a linear filter that includes spatial and temporal lag terms is a sufficient approximation for the more theoretically consistent nonlinear model. The composite alternative is written as:

$$(6) \quad B(s, t) = \mathbf{X}\beta + \varphi_S \mathbf{S}B(s, t) + \varphi_T \mathbf{T}B(s, t) + \left(\frac{K}{1 + de^{T_k}} \right) + \left(\frac{B(s_0, t_0)e^{-(S_k)^2/4\gamma T_k}}{2\sqrt{\pi\gamma T_k}} \right) + v(s, t)$$

where φ_S and φ_T are spatial and temporal lag parameters, respectively and $v(s, t)$ is an iid normal error and (5). Namely, we estimate

each using GMM where the econometric error term, $v(s, t)$ is used to form the moment conditions that allow for the nonlinear estimation of all model parameters.

Spatial-temporal econometric models are often estimated using maximum likelihood (Anselin 1988). Slade (2004), however, notes that maximum likelihood is infeasible for large problems because the dimensions of the spatial weight matrix rise with sample size.² Therefore, alternative methods, such as Generalized Method of Moments (GMM) (Kelejian and Prucha 1998, 1999; Pinkse and Slade 1998; Bell and Bockstael 2000), are becoming increasingly popular. In this study, we follow a two-step approach similar to Cohen and Morrison-Paul (2003) in that we consistently estimate the population equation with GMM in the first step, and then estimate the error process in a second step using maximum likelihood. In the GMM step, the set of instruments includes all exogenous variables in the model, in addition to spatially and temporally lagged values of the dependent variable and all other explanatory variables as suggested by Kelejian and Prucha (1998). The weighting matrix for the GMM procedure is constructed by first estimating the $v(s, t)$ vector with an instrumental variables method (two-stage least squares). This ensures that the GMM algorithm produces consistent parameter estimates and, hence, consistent estimates of ϵ for the next stage. Within the GMM framework, the set of instruments consists of all exogenous variables and combinations of $\mathbf{S}B(s, t)$ and $\mathbf{T}B(s, t)$ as suggested by Kelejian and Prucha (1998).

In order to implement this approach, however, it is necessary to first define the elements of the spatial and temporal weight matrices. There are a number of ways in which this can be done, particularly with respect to the spatial matrix. Whereas Pace et al. develop an integrated, spatial-temporal weight matrix that combines spatial and temporal relationships, this approach is only feasible in a linear econometric model. Therefore, we create independent spatial and temporal weight matrices and allow the spatial and temporal relationships among observations to be reflected through the nonlinear structural model. Our approach follows Anselin (2002), who provides a useful taxonomy wherein the elements of the weight

² The spatial weight matrix, \mathbf{W} , for N observations is an $N \times N$ matrix where each element is the distance, in some metric, between the row and column entry. In a spatial context, multiplying a dependent variable by \mathbf{W} creates a spatial-lag of the variable.

matrix are motivated by either ad hoc or theoretical considerations. Because our insect data was gathered in an experimental plot, there were no natural barriers to movement in any direction and all cells were relatively homogeneous. Therefore, we use a direct measure of distance (Euclidean distance) from one sample plot to each of the others, and form weights by inverting and row-normalizing each element of the distance matrix. Temporal distance, on the other hand, is defined simply in terms of one-period lags between one observation and the previous. Even with such simple measures of distance, however, the combined modeling of temporal and spatial effects can potentially cause the model to be excessively complex with only a few neighboring observations.

Stochastic Insect Diffusion

Insect populations grow and spread according to biological processes. As such, observed insect diffusion paths are expected to vary considerably around the deterministic patterns described by the theoretical model developed in the previous section. Sunding and Zivin (2000), Saphores (2000), and Saphores and Shogren (2005) model the diffusion of invasive species as a geometric Brownian motion. Using the composite-model error as an example, the spatial-temporal equivalent of their purely temporal specifications is written as:

$$(7) \quad dv(s, t)/v(s, t) = \mu dt + \eta ds + \sigma_t dz + \sigma_s dr$$

where μ is the drift rate per unit of time, dt , η is the increment per unit of spatial distance, ds , dz is an increment of a standard temporal Wiener process with zero mean and variance equal to dt , dr is similarly an increment of a spatial Wiener process with zero mean and variance equal to ds , σ_t is the standard deviation of the temporal process and σ_s is the standard deviation of the spatial process. As Sunding and Zivin note, the pure-temporal version of equation (7) captures several empirical regularities observed across insect groups. Namely, per-period changes in the population as well as the population itself are normally distributed, population levels are always nonnegative, and short-run dynamics are dominated by the volatility component whereas long-term dynamics are dominated by trend.

It is not likely, however, that any trend away from the mean in (7) is likely to be sustained over the long-run as insect populations cannot grow without bound, nor is it likely that they disappear without some outside influence. Consequently, the process in (7) is modified to include a mean-reversion term so that:

$$(8) \quad dv(s, t)/v(s, t) = \kappa(\overline{v(s, t)} - v(s, t)) dt + \eta ds + \sigma_t dz + \sigma_s dr$$

where κ is the rate of reversion to the mean. Further, insect populations are also subject to periodic “spikes” or periods of rapid growth driven by environmental factors that are otherwise not accounted for in the model. We model these instances as jumps in the stochastic process for $v(s, t)$ (Merton 1976; Jorion 1989; Naik and Lee 1990), so the most general form of the error process becomes:

$$(9) \quad dv(s, t)/v(s, t) = (\kappa(\overline{v(s, t)} - v(s, t)) - \lambda\phi) dt + \eta ds + \sigma_t dz + \sigma_s dr + \phi dq$$

where jumps occur according to a Poisson process q with average arrival rate λ and a random percentage shock, ϕ . The random shock, in turn, is assumed to be log-normally distributed with δ^2 . The Poisson process q describes a random variable that assumes a value of 0 with probability $1 - \lambda$ and 1 with probability λ .

Because equation (7) defines the error process in both s and t , it implicitly defines two error processes—one in space and the other in time. Whereas Cohen and Morrison-Paul (2003) and Hsieh, Irwin, and Forster (2005) treat the error process from the spatial autoregressive model as following first a spatial process and then a temporal one, this is somewhat arbitrary as the two processes in fact occur simultaneously. Focusing on one dimension while holding the other constant, however, allows us to specify and estimate stochastic processes in both space and time from a single starting point such as (7). For example, in this equation the parameter η is interpreted as the change in the deviation from mean population, holding all temporal influences constant, that is: $dv/ds|_{dt, dz, dq} = v\eta$. On the other hand, the temporal drift, jump and reversion parameters are interpreted the same way, that is, they represent the incremental

variation per unit of time for a given spatial observation: $dv/dt|_{ds} = v((\kappa(\overline{v(s, t)} - v(s, t)) - \lambda\phi) + \sigma_t dz/dt + \phi dq/dt)$. In the empirical application, we estimate both of these functions separately in order to identify the stochastic temporal and spatial parameters in a consistent way.

Estimates of (9) are obtained by maximum likelihood estimation over the entire sample data set, using the likelihood function:

$$(10) \quad L(v|s_0) = -T\lambda - \frac{T}{2} \ln(2\pi) + \sum_{t=1}^T \ln \left[\sum_{n=0}^N \frac{\lambda^n}{n!} \frac{1}{\sqrt{\sigma_t + \delta^2 n}} \times \exp \left(\frac{-((dv/dt)/v - \kappa(\bar{v} - v) - n\phi + \sigma_t/2 + n\delta^2/2)^2}{2(\sigma_t + \delta^2 n)} \right) \right]$$

and a similar expression for the dv/ds process. In (10) we approximate the change of dv/dt with a discrete change: $dv(s_0, t) = v(s_0, t) - v(s_0, t - 1)$ holding s constant at s_0 and dv/ds in a similar way, holding t constant at t_0 : $dv(s, t_0) = v(s, t_0) - v(s - 1, t_0)$. In the next section, we show how parameter estimates from (10) are then used to forecast insect population values and, hence, determine equilibrium prices for insect derivatives.

Risk-Neutral Derivative Pricing

Insect derivatives are contingent securities. There are five essential elements that form any insect derivative: (1) the underlying insect population index, (2) the length of time of the contract prior to expiration, (3) the location for where the underlying insect population is reported (e.g., farm, orchard, experiment station or larger aggregation of farms), (4) the dollar value attached to each unit of the underlying index (marginal loss in revenue attributable to an additional insect), and (5) the strike population index value. At the agreed expiration date of the option, a holder of a call option will receive payment if the insect population index is greater than the strike level, and the holder of a put option will receive payment if the insect population index is less than the strike price. The amount of payment is equal to the level of insect populations that are greater (less) than the strike level multiplied by some notional dollar value per unit of the underlying insect population index. If the option is not exercised, the option buyer will forfeit his option premium. Sellers of options, or option writers,

receive a premium for providing this option to the option buyer.

Theoretical Insect Derivative Pricing Model

Proper pricing of such an instrument is critical for its successful trade. If the insect population represents a hedgeable risk, or one that growers can transfer by trading an underlying futures contract, then it would be possible to price an insect option using a traditional, no-arbitrage, Black-Scholes pricing model. However, as in the case of weather derivatives (Richards, Manfredo, and Sanders 2004), insects are not tradable assets. Without an effective hedge, it is necessary to consider the market price of risk and devise a way of estimating its impact on derivative prices.

We account for the market price of risk using the risk-neutral valuation model of Cox, Ingersoll, and Ross (1985). Applying this model involves a three-stage algorithm. First, the insect population process must be reduced to a martingale, Q , (essentially, a zero-drift stochastic process) by estimating the distribution governing insect diffusion as described above and removing all systematic components from the observed process in space and time. This step—“risk neutralizing” the process—means that the best guess of the insect population at time t_1 is its value at t_0 , or: $E[B(s_0, t_1)] = B(s_0, t_0)$. By removing the predictable components of each part of the insect process, we change the Weiner processes dz to dy and dr to dp , where y and p are Q -Weiner processes (Alaton et al. 2002). The second step consists of forming an expectation of the intrinsic value of the derivative under the Q measure defined by the risk-neutralized process. In the third step we discount the expected payoff value back to the current date at the risk-free rate. This discounted expected payoff is the market equilibrium price of the derivative.³

More formally, given a constant market price of risk, the martingale that defines total (deterministic and random) time-variation in the underlying index becomes:

$$(11) \quad dB(s_0, t)/B(s_0, t) = dB^m(s_0, t)/B^m(s_0, t) + (\kappa(\bar{v} - v) - \lambda\phi - \psi_t \sigma_t) dt + \sigma_t dy + \phi dq$$

³ Clearly, discounting is not necessary for a derivative defined between two points in space.

where dy is now a Q -Wiener process and ψ_t is the market price of risk, expressed on a per unit basis. Unlike other types of derivatives, instruments written on fugitive resources of any type (insect, wildlife or bird populations, water, air pollutants, and potentially many others) must reflect stochastic variation in the spatial dimension as well. Although the temporal notion of an expiry date is more intuitive to those familiar with traditional derivative securities, the analogy to a spatial derivative is valid, but more complex. If the value of the underlying index varies in a random way from one place to another, it is possible to define a derivative security from the perspective of one point with respect to any other point. Similar to the temporal case, it is important that the spatial stochastic process be stable so that the expected value of the difference between populations at the base and reference locations depend only on the distance between them and not on the physical location. This is a critical assumption, but is likely to be violated in cases where there are other impediments to the movement or interaction of insect populations such as streams, hedges, or wind-breaks. In these cases, the spatial distance matrix can be modified in order to address these discrete measures of biological distance. Specifically, by removing all systematic variation in s , we define a spatial martingale similar to the temporal version in (11):

$$(12) \quad dB(s, t_0)/B(s, t_0) = dB^m(s, t_0)/B^m(s, t_0) + (\eta - \psi_s \sigma_s) ds + \sigma_s dp$$

where ψ_s is the market price of spatial risk per unit of B . With these two functions, we can then use the parameters estimated above to find the expected insect population at an "expiry" value of t and of s , given values for each market price of risk. As in the case of weather derivatives, however, finding reliable estimates of the market price of risk represents a significant empirical problem (Cao and Wei 1999; Alaton, Djehiche, and Stillberger 2002; Richards, Manfredo, and Sanders 2004).

Typically, researchers attempt to calibrate the market price of risk using price series from similar instruments that are traded on organized exchanges. For insect derivatives, however, no such exchange exists. It is a basic tenet of asset pricing that a portfolio of two derivatives written on B can be constructed such that their combined return is equal to the risk-free

rate. Thus, if we define the rate of drift in (11) as $\mu = dB^m/B^m + (\kappa(\bar{v} - v) - \lambda\phi)$, the return to the risk-neutralized process must be equal to the risk-free rate: $\mu - \psi_t \sigma_t = r$. Using any asset pricing model—the discrete-time capital asset pricing model (CAPM) for example—it must also be the case that the return to any particular asset must be equal to the risk-free rate plus a security-specific market-risk premium: $\mu = r + \beta(r_m - r)$, where r_m is the return to the market portfolio, and β measures the systematic risk of the security. In the CAPM, however, we know that $r_m - r = \psi_t$ so the risk premium to any asset is determined by the market price of risk and the security-specific measure of systematic risk. Systematic risk, in turn, depends on the covariance of asset and market returns and the variance of market returns: $\beta = \sigma_{BM}/\sigma_m^2$, so any security with returns that are statistically independent of the market must have a zero market price of risk. Because this is indeed likely to be the case for localized insect populations, we set $\psi_t = 0$ in (11) and calculate the equilibrium price by discounting the expected terminal value of the derivative at the risk free discount rate.

Empirical Option Pricing Model

The theoretical framework described in the previous section is used to price a complete chain of spatial and temporal insect derivatives for the *B. tabaci* data. Unlike financial options where the underlying index is driven by temporal variation, an insect option is defined over both temporal and spatial changes in the underlying index—in this case a population count of *B. tabaci* on cotton plants. Because space has two dimensions and time has one, the chain we estimate is perhaps more accurately described as an option "cube" for a discrete set of expiry dates and potential insect locations.

To account for the spatial dimension, a pricing grid is constructed for each point in time, corresponding to the experimental plots where the population data were collected, which consist of five rows and five columns.⁴ The grid is numbered horizontally and vertically from one to five where point $s_0 = (1,1)$ is considered the origin of the insect infestation. Distance is defined in Euclidean terms, consistent with

⁴ Note that this design can easily accommodate insect population data gathered in a nonexperimental environment. Selection of the grid design is, however, up to the researcher's discretion.

the model developed above. An infestation is assumed to commence at s_0 and then spread according to Fick's Law. Therefore, the spatial mean of the insect population at any given point on the spatial grid is $B^s(s, t_0)$ following equation (4). The temporal mean, $B^t(s_0, t)$, is governed by the process described in equation (3). Therefore, the mean level of insects at each point in space on the spatial grid is $B^m(s, t)$, which is the sum of the spatial and temporal mean for a particular point in time t . The stochastic temporal component is modeled following the martingale defined in equation (11) and the stochastic spatial component modeled as in equation (12).

Considering both the mean and stochastic components, the number of total insects at each point in space and time, $B(s, t)$ is:

$$(13) \quad B(s, t) = B^m(s, t) + dB(s_0, t) + dB(s, t_0)v$$

where $dB(s_0, t)$ is the instantaneous change from equation (11) and $dB(s, t_0)$ is the instantaneous change from equation (12). Therefore, each cell in the spatial grid has a different insect population value.

Given the underlying index and the time to expiration, the other elements needed to price an option on insect derivatives is the designated strike population level, the risk-free rate, and the notional value of the derivative. In practice, the strike level used should coincide with economic loss threshold levels associated with a *B. tabaci* infestation, or levels of infestation which lead to increased and costly eradication efforts.⁵ For illustrative purposes, we use the following strike levels in estimating the option prices: 5, 10, 20. We assume that the notional value of each insect is one dollar and the risk-free rate of interest, r , is 5%, which is reflective of short-term interest rates in the fall of 2006. However, it is important to note that the choice of the risk-free interest rate does not materially affect the value of the option.

In the absence of a closed-form solution to the option pricing problem, Monte Carlo simulation procedures are used to estimate the fair value of call options at each strike level for times to expiration of 1, 2, and 3 months. Monte Carlo simulation has been used extensively in the literature in valuing options as it

is an effective and easily generalizable way to value an option where the underlying index follows a complex process. The steps in the Monte Carlo simulation are as follows. First, the temporal Q -Wiener process in equation (11), dv , is specified as $\epsilon_t \sqrt{t}$ where $\epsilon_t \sim N(0, 1)$ and t is the time to expiration of the option expressed in days. Second, the spatial Q -Wiener process in equation (12), dp , is $\epsilon_s \sqrt{s}$ where s is the normalized inverse Euclidian distance defined in the spatial weighting matrix S for a particular point on the spatial grid and $\epsilon_s \sim N(0, 1)$. The jump diffusion process described in equations (11) and (12) is also modeled within the same Monte Carlo algorithm, where the two stochastic elements of the jump diffusion process are the arrival rate and the distribution of the random shock. Hence, for a given time to expiration t , a Monte Carlo simulation is run using 10,000 draws from the distributions ϵ_t and ϵ_s , the distribution governing the arrival rate of the jumps in the jump diffusion process, and the distribution of the random shock. The Monte Carlo simulation produces a distribution of option payoff values as expressed in equation (13). The mean of the payoff distribution is then discounted back to the present by the time to maturity t using rate r yielding the option value. Therefore, in general, the value of the call option at a given point in space s for expiration at time t and strike level x , $C(s, t)$, can be expressed as:

$$(14) \quad C(s, t) = e^{-rt} \int_x^\infty f(B(s, t)) \times (B(s, t) - x) dB(s, t)$$

where the integral is approximated using the Monte Carlo algorithm.

Risk Simulation Model

The discussion to this point has made the case that designing and pricing an insect derivative is indeed possible, but whether trading them is desirable from an economic standpoint depends on their effectiveness in improving risk/return outcomes for agricultural producers. We define effectiveness in terms of expected utility—whether hedging with insect derivatives is likely to improve returns at given levels of expected net income volatility. As is standard practice, utility rises in the level of net income, but at a declining rate (declining marginal utility of income). Define the degree of risk aversion of the representative grower as

⁵ "Economic loss thresholds" are concepts in entomology that refer to the population level at which a grower can reasonably predict that the population will rise to the "economic injury level (EIL)" or the level at which control costs are at or below crop losses.

γ , where higher values of γ mean that marginal utility falls at a greater rate as income rises. Further, define a power utility function where $U[g(\pi)]$ represents the utility of a representative grower g from earning an uncertain level of profit such that:

$$(15) \quad E[U_g(\pi)] = E[\pi_g^{1-\gamma}/(1-\gamma)]$$

where γ is the coefficient of risk aversion, $0 < \gamma < 1$ for concavity and $E[\]$ is the expectation operator. Power utility is an attractive alternative because it is a simple representation that possesses all the characteristics required of a well-behaved utility function: it is concave by construction, it implies a constant relative risk aversion level in profit provided the coefficient of risk aversion is bound on $[0, 1]$ and exhibits decreasing absolute risk aversion as wealth rises. If the coefficient of risk aversion is equal to zero, then the grower is “risk neutral,” or indifferent to the volatility of his or her income stream.

Defining the objective in terms of expected utility allows us to estimate the risk premium associated with a given control strategy. Specifically, define the certainty equivalent (*CE*) value as the dollar amount that he or she would accept with certainty in lieu of the risky prospect of receiving an uncertain amount of net income with expectation $E[\pi_g]$. In a power utility framework, a grower’s *CE* value is found by solving (15) for π_g . The risk premium grower g is willing to pay, therefore, becomes:

$$(16) \quad R(\pi_g) = E[\pi_g] - CE(\pi_g) \\ = E[\pi_g] - ((1-\gamma)E[U_g])^{1/(1-\gamma)}$$

for an uncertain level of net income. For the *B. tabaci* example, net income from growing cotton is assumed to be equal to the difference between cotton revenue and total production cost, where revenue is the product of uncertain yields (q_g) and prices (p): $\pi_g = pq_g - K(c_g)$ and total production cost depends on the level of insect control activities, c_g . In order to capture the likely diminishing marginal returns to insect control activities, yield in year t is assumed to be a simple Cobb–Douglas (log-log) function of insect density, control activities and a binary variable to account for year-specific population differences:

$$(17) \quad \ln(q_{g,t}) = \beta_0 + \beta_1 \ln(B_{g,t}) + \beta_2 \ln(c_{g,t}) \\ + \beta_3 D94 + \varepsilon_{g,t}$$

where $D94$ is a binary variable for the year 1994 ($D94 = 1$ if the year is 1994 and is zero otherwise), $\varepsilon_{g,t}$ is a grower-specific i.i.d. random error vector and the remaining variables are as described above. Without data on other inputs, the yield function in (17) assumes all growers use best-practice technology so that β_0 represents their average yield, conditional on optimal input application. Further, insect control and, hence, populations are assumed to be endogenous, so equation (15) is estimated using an instrumental variables procedure (two-stage least squares) where the set of instruments includes all exogenous and pre-determined variables in the system.⁶

We evaluate the risk-return performance of alternative hedging strategies using three commonly used metrics: (1) a Sharpe ratio, (2) a 5% Value-at-Risk (VaR) measure, and (3) a certainty equivalent value. The Sharpe ratio is a measure of return per unit of risk derived from an expected utility maximization framework. Specifically, it is defined as the ratio of excess returns to an asset to the coefficient of variation of its returns, where “excess returns” are defined relative to the risk-free rate of return. Formally, the Sharpe Ratio is written as:

$$SR_g = (R_g - R_f)/(s_g/\bar{R}_g)$$

where R_g is the return to the asset or venture in question, R_f is the risk-free rate of return, s_g is the coefficient of variation of returns and \bar{R}_g is the mean return. Value-at-Risk (VaR) measures the maximum amount a firm can expect to lose at a certain confidence level for a certain period of time. For example, if a grower’s VaR is $-\$200.00$ per acre at 5% on an annual basis, this means that there is a 5% chance he or she will lose at least $\$200.00$ during the year. VaR provides a very intuitive notion of the monetary equivalent of the risk facing a firm as it immediately converts a notion of spread or dispersion into a dollar-equivalent figure (Jorion 1997). Third, we compare the *CE* value defined in equation (16) for alternative insect-risk management strategies. From a grower’s

⁶ Input decisions are likely to be influenced by the presence of a risk management tool (Horowitz and Lichtenberg 1993), which would in turn influence the value of the derivative. In the absence of detailed data on chemical input usage, we ignore the likely endogeneity of input decisions in the empirical analysis. Further, Clark and Carlson (1990) show that pesticide resistance causes the absence of insects to be a common property resource. In this paper, derivative prices are calculated based on privately optimal decisions, and not on socially optimal outcomes.

perspective, a higher *CE* value is preferred because it implies a lower “cost of risk” or risk premium that a rational investor would demand. By comparing each of these measures between hypothetical scenarios wherein growers do or do not use insect derivatives, we determine whether bug options represent potentially valuable risk management tools.

A number of assumptions are made in order to implement the insect-derivative simulation model. First, the number of contracts used to hedge insect-yield risk from a typical acre of cotton in the Imperial Valley (the “hedge ratio”) is determined by estimating a simple linear regression of yield on insect densities (Cecchetti, Cumby, and Figlewski 1988) and is estimated at 5.57. The slope parameter in this regression shows the marginal impact of a one-adult-insect-per-leaf rise in population, so multiplying the marginal impact of one insect by the price of cotton provides an estimate of the marginal revenue-loss, or the hedge ratio. Second, in order to determine the independent effect of random insect growth on yields, the simulation is conducted with insect control activities held at their mean. While understanding the role of biological and chemical insect suppression is an important pursuit, the point of this research is to show how financial risk can be mitigated independent of traditional control methods. Third, although cotton prices represent another source of economic risk in reality, prices are fixed at their long-term average. Finally, the coefficient of relative risk aversion (3*b*3) is allowed to vary from 0.1 (near-risk neutrality) to 0.9 (extreme risk aversion) in order to convey the importance of attitudes toward risk in determining the value of insect derivatives in terms of the expected-utility framework. The net income/expected utility model is simulated using Monte Carlo methods with @Risk stochastic simulation software (Palisade Corporation 2001).

Insect Population Data

The data for this study consist of two years of experimental field-trial data on *B. tabaci* population growth and yield damage gathered by researchers based at the Western Cotton Research Laboratory (WCRL) in Phoenix, AZ using cotton fields in Brawley, CA (Naranjo, Chu, and Henneberry 1996). *B. tabaci* is a particularly nefarious pest in the U.S. Southwest as they tend to travel large distances, reproduce quickly and impair yields significantly by

depriving the plant of vital nutrients. Weekly counts of adult *B. tabaci* were collected each year over two seasons (21 weeks in 1994 and 18 weeks in 1995) for 25 different plots, arrayed in a 5 × 5 latin-square design. Within the 25 plots, there are five different treatments, ranging from nothing (control) to a relatively intensive regimen of insecticides. No attempt was made here to estimate the impact of specific chemicals or application techniques, due to their heterogeneity, so control procedures were assigned a linear scale rising from 1 (control) to 5 (intensive chemical management). In this way, we are able to study the impact of more intensive control activities on population levels at different times during the season. Control efforts cause the data to exhibit greater variability than would otherwise be the case, allowing us to more clearly identify the underlying population diffusion process. At both locations, yield samples taken at harvest for each plot provide data regarding the yield-injury relationship in cotton. The data summary in table 1 shows that there is considerable variation in insect numbers in these test plots—from zero to 241 adults per leaf—depending on the sample date and the control activity. While these data show far more population growth than growers experience in real-world settings (due largely to the pervasiveness of insecticide spraying, or other biological control methods) they do provide an accurate assessment of the type of population pressure growers face when they design control strategies. Because these strategies are costly, access to a cost-risk management tool is important.

Table 1. Insect/Weather Summary Statistics

Variable	N	Mean	Std. Dev.	Min.	Max
Treatment (#)	975	3.000	1.414	1.000	5.00
Eggs (#/cm ²)	975	6.830	11.27	0.030	104.54
Nymphs (#/cm ²)	975	1.486	2.715	0.000	25.970
Adults (#/leaf)	975	12.793	27.72	0.000	241.00
Temp. Max. (°F)	975	100.72	10.47	73.00	115.00
Temp. Min. (°F)	975	67.564	10.14	47.00	82.000
CDD (°F)	975	19.538	9.642	0.000	34.000
Rain (in.)	975	0.002	0.016	0.000	0.100
CCDD (°F)	975	1,216.5	853.1	116.0	3,017.0
CRain (in.)	975	1.644	0.294	1.300	1.960
Yield (kg/ha)	50	1,553.0	394.22	660.0	2,380.0

Notes: Variables are as follows: Treatment is the number of pesticide applications per season, “Eggs” is the number of eggs per cm², “Nymphs” are immature insects per cm², “Adults” are adult insects per leaf, “Temp. Max.” is maximum daily temperature in °F, “Temp. Min.” is minimum daily temperature, “CDD” is cooling degree days (H – 65°F), “Rain” is amount of rain received, in inches, on one day, “CCDD” is the cumulative number of CCDs over the sample period, and “CRain” is the cumulative rainfall over the sample period.

All weather data are from the National Oceanic and Atmospheric Agency (NOAA). Daily weather data are used, but are aggregated out to a weekly basis so they are comparable to the experimental insect population data. Insect numbers are expected to be influenced by a number of weather-related variables, including the daily maximum and minimum temperature, the number of cooling degree days (CDD) on a particular day and over the entire growing season, and the amount of rain, again both on a particular day and on a cumulative basis over the growing season.⁷ From the data summary in table 1, it is apparent that there is little variability in rainfall in the Imperial Valley over a typical growing season, but temperature varies over a considerable range. Consequently, the results in this study should be interpreted as representative of a relatively extreme climate, one with little rainfall but more temperature variability than in most other growing locations. The effect of these temperature extremes will be determined in the empirical growth model results that follow.

Data for the risk management simulation model are taken from representative Imperial County cotton farm budget prepared by University of California Cooperative Extension officials (University of California 2005). Operating costs reflect all land preparation, seeding, growing and harvesting costs and are expressed in current, 2004 values. Growing costs include the material and labor cost for a number of insect treatments equal to the sample average from the *B. tabaci* trial data. Revenues, on the other hand, are calculated using 2004 harvest prices obtained from the Economic Research Service, USDA (USDA 2005b).

Results and Discussion

The empirical model described above necessarily involves a number of steps. Therefore, our discussion of the results considers each in turn: (1) estimates of the mean insect growth function, (2) estimates of the process governing stochastic variation from the mean, (3) estimates of the *B. tabaci* damage function, and (4) price estimates of an example insect derivative, defined as a call option on *B. tabaci* at the

Brawley, CA research station. Although the primary objective of this study is to develop a pricing model for insect derivatives, no empirical research to date treats insect population growth and diffusion in an integrated spatial-temporal framework, so the results from steps (1) and (2) are likely to be of some interest.

While Fick's Law and the logistic growth model impose some structure on the nonlinear diffusion model, a linear or composite model may provide a superior fit. Therefore, finding the best model becomes a statistical exercise. To find the best specification, we estimated three different nonlinear spatial-temporal models. In order to maintain a consistent estimation framework, all models were estimated using the same set of instruments, which were described above. Within each class of models, specification tests are based on prediction accuracy, or a mean-square error (MSE) criteria and on a quasi-likelihood ratio (QLR) testing procedure (Davidson and MacKinnon 2004). In each case, the QLR test statistic value is $G = (Q_2 - Q_1)$ where Q_2 is the GMM objective function value under the alternative hypothesis, and Q_1 under the maintained. G is chi-square distributed with q degrees of freedom, where q is the number of parameters that differ between the two models.

In table 2, we report the estimation results from three alternative specifications: (1) only temporal-growth effects, (2) spatial-diffusion and logistic growth, and (3) a composite model of spatial-temporal growth and diffusion and linear spatial-temporal lags. Comparing models (1) and (2), the temporal growth model outperforms the spatial-temporal model in terms of predictive accuracy, and explanatory power, but produces a slightly higher Q value. With one degree of freedom, however, the difference (2.200) is not statistically significant, although the spatial diffusion parameter is statistically significant on its own. Clearly, adding a spatial diffusion term does little to improve model fit, but we cannot reject the theoretical motivation for including it. Comparing both models to the composite alternative, however, produces a significantly better fit to the data. In terms of predictive accuracy, the results in table 2 show that composite, nonlinear spatial-temporal growth and diffusion model produces a MSE value less than half of either of the other two. This is perhaps to be expected given the larger parametric size of the composite model. The composite model also produces a far higher R^2 value and a G statistic value

⁷ A cooling degree day is defined as the difference between the average temperature on a given day and 65 °F, or $CDD = (\bar{H} - 65^\circ F)$, where \bar{H} is the average temperature on a given day.

Table 2. Nonlinear Spatial-Temporal Insect Model: GMM *B. Tabaci*, Imperial Valley, CA 1994–95

Var.	Temporal		Spatio-Temporal		Composite	
	Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.
α	0.293*	0.087	0.293*	0.088	0.269*	0.106
γ	–	–	0.113*	0.018	0.197*	0.099
SY	–	–			1.223*	0.047
TY	–	–			0.454*	0.032
STY	–	–			–0.522*	0.060
Treat	–0.283*	0.047	–0.283*	0.049	–0.079*	0.037
CDD	0.106*	0.006	0.106*	0.007	0.009	0.006
Rain	–6.845	5.141	–6.846	5.095	–1.559	3.466
1994	0.598*	0.164	0.598*	0.163	0.064	0.113
R^2	0.163		0.164		0.629	
MSE	6.387		6.388		2.838	
Q	2,172.382		2,170.182		1,795.073	

Notes: A single asterisk (*) indicates significance at a 5% level. γ is defined as the spatial diffusion parameter and α is the temporal-growth rate. Q is the value of the GMM objective function.

of 364.999. With three restrictions at a 5.0% level of significance, the critical value for the QLR (chi-square) test is 7.815, so we easily reject the simple spatial-temporal model in favor of the composite. Note, however, that among the other explanatory variables only the treatment effects are statistically significant in the composite model, whereas temperature and the 1994 dummy are not significant. This result suggests that failing to completely account for spatial and temporal lags leads to an overestimate of the effect of environmental and control factors. Or, this may also reflect that the temperature simply does not vary enough in the Imperial Valley to have an appreciable effect on insect growth rates. Estimates of the key structural parameters—measuring growth (α) and diffusion (γ)—are, however, similar among all models.

Based on the goodness-of-fit results reported in table 2, we use the errors from the composite model to estimate the stochastic process underlying insect diffusion. Four models are considered for this purpose, three temporal and one spatial. Each model is estimated using maximum likelihood. Among the temporal models, we adopt a “simple-to-general” model selection strategy in which successive variations include elements that are likely to be important to the noise process driving insect growth. Tests of the regression residuals left by each of the models described in table 2 (Jarque and Bera 1987) reject the null hypothesis of normality. Preferred models are chosen on the basis of a likelihood ratio (LR) testing procedure, the results of which are shown in table 3. The LR test statistics show that each model is preferred to a null model (in

Table 3. Stochastic Insect Error Process: MLE *B. Tabaci*, Imperial Valley, CA 1994–95

Var.	GBM		MR-GBM		MR-GBM-J		S-GBM	
	Est.	Se	Est.	Se	Est.	Se	Est.	Se
μ	0.533*	0.162	–	–	–	–	–	–
σ	274.481*	15.547	274.762*	17.752	1.335*	0.033	1.789*	0.042
κ	–	–	0.006*	0.002	0.016*	0.003	–	–
δ	–	–	–	–	795.720*	132.561	–	–
ϕ	–	–	–	–	399.731*	66.423	–	–
λ	–	–	–	–	0.422*	0.021	–	–
η	–	–	–	–	–	–	0.002*	0.0006
LLF	–3,909.400		–3,901.877		–2,366.719		–1,850.298	
χ^2	52.938		68.016		3,216.254		5,236.616	

Notes: GBM is a geometric Brownian motion, MR-GBM is a mean-reverting GBM, MR-GBM-J is a mean-reverting GBM with Poisson jumps. The chi-square statistic is calculated as $\chi^2_q = 2(LLF_U - LLF_R)$ where LLF_U is the unrestricted log-likelihood value and LLF_R is the log-likelihood value of the null model and q is the number of restrictions. A single asterisk indicates significance at a 5.0% level.

Table 4. Call Option Prices: $X = 5$ Insects

	Grid Location				
	1	2	3	4	5
<i>t</i> = 1 month					
1	8.832	8.266	6.694	4.475	2.053
2	8.289	7.718	6.201	4.087	1.771
3	6.701	6.2224	4.921	3.009	0.987
4	4.466	4.095	3.008	1.480	0.082
5	2.043	1.758	0.967	0.076	0.000
<i>t</i> = 2 months					
1	18.506	18.010	16.577	14.438	11.776
2	17.979	17.536	16.148	13.997	11.405
3	16.590	16.134	14.857	12.871	10.418
4	14.435	14.032	12.849	11.075	8.873
5	11.773	11.385	10.411	8.858	6.973
<i>t</i> = 3 months					
1	20.438	20.106	19.051	17.405	15.345
2	20.059	19.743	18.728	17.122	15.063
3	19.041	18.737	17.757	16.209	14.235
4	17.447	17.112	16.237	14.793	12.957
5	15.306	15.040	14.217	12.921	11.229

Notes: Option values are found using Monte Carlo simulation with $T = 10,000$ and $r = 5.0\%$.

which all parameters are restricted to zero), but also that the most comprehensive model is preferred to the other two. Consequently, we choose a mean-reverting geometric Brownian motion model with Poisson jumps to generate temporal insect forecasts. Only one spatial model was considered because there is no a priori reason to expect any spatial mean reversion nor discrete jumps. Compared to a null model, however, the results in table 3 show that this simple spatial error specification provides a satisfactory fit to the data. Therefore, the full model in equation (9) above is used to forecast both spatial and temporal insect population variation and to calculate all derivative prices.

The resulting prices are shown in table 4 for a strike population value $x = 5$.⁸ The prices shown in this table describe one part of a complete “chain” of options, consisting of three different times to maturity for one strike. If an analytical option pricing formula were available, it would be possible to derive “greeks” for the options, or the sensitivity of prices to changes in each of the underlying parameters. Because we use numerical solution methods, however, this table shows one greek, the theta, or sensitivity to time to expiry. Comparing prices

across different population values is straightforward and produces a second, and important, greek called the “delta.” Delta is important because it determines the number of options that must be purchased in order to offset a particular level of insect risk.

Relative to conventional option pricing methods, the option values in table 4 reveal the importance of valuing the spatial component of insect diffusion. For every time to expiry, option values decrease the further the spatial distance from the origin (1,1), as expected. This is consistent with Fick’s Law. In fact, the differences in option values are quite substantial across the grid. For example, with one month to expiry the value of the call option at the origin (1,1) is \$8.832, while it is \$0.00 at the opposite corner (5,5). Consistent with option pricing theory, as the time to expiry increases, so does the value of the option. This is best illustrated by looking at the average option value over the entire grid for each time to expiry. For one month to expiry, the average option value is \$3.929, for 2 months is \$13.516, and for 3 months is \$16.652. Also as expected, given the nature of Fick’s Law, the differences between option values within the grid also change depending on the time to expiry. As the time to expiry increases, insects, in theory, have more time to disperse along the grid. At 1 month to expiry, the difference between the (5,5) and (1,1) options is 100.00% of the (1,1) value, whereas it is only 45.1% at 3 months to expiry. The spatial distribution of prices, therefore, suggests that trading a chain of insect derivatives—more appropriately referred to as a “grid”—would be necessary to hedge insect risk.

Similar patterns arise with strike values of 10 and 20 insects. As expected, higher strike values lead to lower option prices. This is expected when valuing call options as raising the strike value reduces the chances that the underlying index will exceed the strike, thus reducing the value of the option.

Traders in this market, however, are likely most interested in the risk management implications from trading insect derivatives. Two risk-metric comparisons are necessary to determine whether insect derivatives will be effective spatial-risk management tools: (1) a “complete” hedging strategy versus no hedging, where complete is defined as a full grid of options, and (2) hedging with no spatial differentiation versus hedging with a full grid of spatially differentiated options. Using the stochastic simulation framework described above, we

⁸ Option prices for strike population values of 10 and 20 insects show a similar pattern and are available from the authors.

Table 5. Risk-Return for Hedging vs. No Hedging

	$\gamma = 0.1$	$\gamma = 0.5$	$\gamma = 0.9$
No Hedging			
Sharpe ratio	1.428	1.428	1.428
VaR (5.0%)	-6.227	-6.227	-6.227
Certainty equivalent	51.223	44.241	21.550
Full Hedging			
Sharpe ratio	2.959	2.959	2.959
VaR (5.0%)	32.056	32.056	32.056
Certainty equivalent	52.282	52.282	52.282

Notes: Monte Carlo simulations conducted with 10,000 draws from standard normal error distributions. See text for assumptions regarding net income calculations.

Table 6. Risk-Return for Spatial vs. Nonspatial Hedge

	$\gamma = 0.1$	$\gamma = 0.5$	$\gamma = 0.9$
Nonspatial			
Sharpe ratio	1.366	1.366	1.366
VaR (5.0%)	-34.031	-34.031	-34.031
Certainty equivalent	54.567	38.734	5.487
Spatial			
Sharpe ratio	2.889	2.889	2.889
VaR (5.0%)	32.058	32.058	32.058
Certainty equivalent	52.400	51.348	50.367

Notes: Monte Carlo simulations conducted with 10,000 draws from standard normal error distributions. Nonspatial assumes all payouts fixed at (1,1) grid location values.

created a representative farm consisting of four separate “fields” consisting of four corners of the pricing grid shown in table 5. In this way, we capture the qualitative implications of a spatial-temporal hedge while creating the most parsimonious modeling framework. A hedge consists of purchasing a number of call options (given by the hedge ratio) in the spring and holding until harvest, whereupon the option is sold at market value. All revenue and cost estimates are based upon the representative farm described in University of California.

Table 5 provides a comparison of the three risk-return measures described above for scenario (1), hedging versus no hedging. Clearly, hedging dominates by all measures. The Sharpe Ratio for hedging producers is more than double that for nonhedging producers, while the VaR is over \$38.00/acre higher. Neither of these measures, however, allows for the likelihood that growers differ in their attitudes toward risk. For growers who are slightly risk averse, hedging provides modest improvements in CE (\$1.06/acre), while for extremely risk averse growers, the improvement is more dramatic: \$28.71/acre, or an amount equal to the mean of hedged net income. How much of this benefit derives from the spatial hedge and how much from a traditional temporal hedge, however, remains an open question.

Table 6 compares a traditional hedge accounting for temporal uncertainty with a spatial-temporal hedge. To make the comparison valid, we define a nonspatial hedge as one in which there is no difference among each of the four “plots” that comprise the representative farm. Therefore, each option is priced, and pays off, to reflect the risk profile at grid location (1,1) whereas the yield outcome is

determined by the actual risk faced at each location on the farm. Admittedly, this scenario builds error into the alternative hedging strategy, but the key question is whether correcting this error is worth the extra effort. According to the results in table 6, accounting for differences in spatial insect risk is indeed important. As in the previous case, the spatial hedge dominates the nonspatial hedge in terms of the resulting Sharpe Ratio, VaR and, for moderate and extreme levels of risk aversion, the CE measure. In fact, the advantage of a spatial hedge over a nonspatial hedge appears to be greater than the advantage of hedging versus not hedging. This result suggests that the spatial basis risk inherent in insect derivatives, which was expected to be severe a priori, is more important than temporal risk of growing insect populations. This is somewhat surprising, but apparent from the growth pattern of insects between 0 and 60 days. The difference in population levels from one part of the grid to the other is far greater than the difference between months for the same grid location.

Conclusions and Implications

This study describes a new class of financial instruments that have the potential to help farmers transfer some of the risk due to damage caused by invasive species, or to offset some of the additional cost in controlling an infestation—insect derivatives. Insect derivatives are contracts that specify a payment from one counterparty to another should an index of insect population density at an agreed location exceed (or fall short of) an agreed level

by a specified expiry date. In this study, we address the primary obstacle to the development of an active trade in insect derivatives, namely the lack of a transparent method of arriving at equilibrium prices. We also develop a new method of accommodating the spatial basis risk that is endemic to a wide range of "real" derivative contracts such as weather options, catastrophe bonds and insect derivatives.

Our pricing model has at its core an insect population forecasting model that synthesizes principles from entomology and spatial econometrics. Under risk-neutral measure, the price of an insect derivative is calculated as the present value of its intrinsic value at expiry calculated at the risk-free interest rate. Because we estimate stochastic processes for insect growth and diffusion in both temporal and spatial dimensions, derivative prices are calculated with reference to a base time period and location. In this way, users will be able to price two-dimensional derivatives. This development has implications beyond the insect derivative application described here. Previously, geographic basis risk in futures contracts or options was simply absorbed as a cost of using derivatives to hedge, or at least managed as part of a well-informed hedging strategy. By pricing basis risk, however, it is possible to hedge changes in an underlying index that may arise both with the passage of time and distance. Weather derivative traders, agricultural commodity futures traders, or anyone else who may be subject to geographic basis risk can benefit from applying this technique to their particular circumstance, given a properly functioning over-the-counter market in the necessary contracts.

Estimation results, obtained using experimental data on whiteflies in California cotton, show that a composite linear/nonlinear spatial-temporal model of insect growth and diffusion is preferred for forecasting the deterministic component, while a mean-reverting geometric Brownian motion process with discrete jumps provides the best fit to the stochastic part. Therefore, the most comprehensive specification is used to forecast spatial and temporal variation in insect population numbers and thereby, input for the derivative pricing model.

Calculating derivative prices involves a simple risk-neutral procedure. We use Monte Carlo simulation methods to calculate derivative prices over a complete "grid" that simulates a grower's farm, each part with a different level of insect risk. Derivative prices are found to vary significantly both over time and over

space as the insect population grows and distributes itself throughout space. It is at least theoretically possible, therefore, that these derivatives could be used to hedge the risk from the spread of an insect species through different parts of a grower's farm. The extent of the improvement in risk-return outcomes that may be possible by hedging with insect derivatives is estimated by simulating three risk-return measures under hedging and no-hedging scenarios on a representative California cotton farm. Because the hedging results dominate the nonhedging measures for this case-study, insect derivatives may indeed be a useful part of an insect-risk management strategy.

By developing a relatively straightforward method of pricing bug options, this research may help growers trade insect derivatives with natural counterparties (for example, chemical companies, insurance companies or nurseries), thereby transferring risk to someone more willing to bear it. Second, by trading insect derivatives, chemical companies may have access to another means of raising capital for new product development, or a way to smooth revenue streams from limited-use chemicals. Third, with an active insect derivative market, growers could use the proceeds from writing options to offset the cost of higher insecticide costs. More generally, this paper provides a method of trading spatial risk. Transportation costs for grain farmers, spatial inequities in demand fluctuation for energy producers or supply shocks in feed supply for cattle ranchers all give rise to instances where basis risk may be important. Trading "spatial options" may be an important development in risk management in many contexts beyond that considered here.

Because of the potential value of insect derivatives, and the weakness of existing data, there are many avenues for future research on this topic. First, the primary limitation of our study is the use of experimental data. While these are currently the only spatial data available on this type of insect, new initiatives in creating GIS maps for *Lygus* bugs and *B. tabaci* are underway (Carriere et al. 2006). Research efforts like this, and others in this vein, will greatly improve our ability to conduct spatial-temporal insect diffusion estimation. Second, our data describe two seasons of movements for one insect species consisting of 39 pooled time series, cross-sectional observations of the 25-cell experimental grid. Although sufficient for the purposes outlined here, practical implementation of insect derivatives would likely

require a longer time series for more spatial locations. It is also important to note that our data are relevant to the population of *B. tabaci* in 1994 and 1995 so our results are not intended to be useful for a trading program initiated in the current year. Resistance, genetic evolution, or the growth of predator populations all mean that trading will have to be based on estimates from contemporaneous data.

The possibility of moral hazard must be taken into account both in crafting insect derivative contracts and in gathering data for index construction. As in the case of weather derivatives, an objective measuring site or methodology can be specified in the contract that removes the insect count from any influence by the grower. In a real-world application, insect derivatives would be defined over relatively wide geographic areas—far wider than the experimental plot used here. Emerging research in entomology (Carriere et al. 2006) applies GIS data handling techniques to create more detailed spatial maps of insect movements, over wider areas, than is currently available. By averaging insect counts over a number of different assessment sites or by creating statistical projections for each cell within a geographic area (using kriging or similar spatial interpolation techniques) the moral hazard problem can be successfully avoided. In fact, the empirical method presented here may be used to determine the appropriate level of spatial aggregation used to construct the population index. This level of aggregation would balance the cost of basis risk with the benefit of removing any possibility for individual growers to tamper with the measurements.⁹ Finally, the novel nature of insect derivatives raises obvious questions as to their usefulness and practicality as risk management tools. However, with the rapidly expanding use of tradable permits to control many other types of externality, both environmental and otherwise, insect derivative trading now seems a natural approach to complete a previously missing market.

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⁹ We thank an anonymous reviewer for suggesting this idea.

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